Elliptic Function Compactons in a class of Generalized Korteweg-DeVries Equations

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Abstract

We study the class of generalized Korteweg-DeVries equations derivable from the Lagrangian:

\[ L(l, p) = \int \left( \frac{1}{2} \varphi_x \varphi_t - \frac{(\varphi_x)^l}{l!(l-1)} + \alpha(\varphi_x)^p(\varphi_{xx})^2 \right) dx, \]

where the usual fields \( u(x, t) \) of the generalized KdV equation are defined by \( u(x, t) = \varphi_x(x, t) \). This class contains compactons, which are solitary waves with compact support. In this paper we obtain the elliptic function compact solitary wave solutions to this class of equations. We prove a theorem that all the solitary wave solutions of the equations of motion obey a simple relationship between the Energy, Momentum and velocity of the solitary wave.

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